Top quark, heavy fermions and the composite Higgs boson

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Abstract

We study the properties of heavy fermions in the vector-like representation of the electro-weak gauge group $SU(2)_W \times U(1)_Y$ with Yukawa couplings to the standard model Higgs boson. Applying the renormalization group analysis, we discuss the effects of heavy fermions to the vacuum stability bound and the triviality bound on the mass of the Higgs boson. We also discuss the interesting possibility that the Higgs particle is composed of the top quark and heavy fermions. The bound on the composite Higgs mass is estimated using the method of Bardeen, Hill and Lindner [1], 150GeV$\leq m_H \leq$ 450GeV.

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Enormous efforts have been made in searching for physics beyond the standard model but up to now a crucial, direct experimental indication is still illusive. One of the most important motivation to study the property of heavy fermions above the energy scale accessible by current accelerators is to look for extra building blocks of nature beyond the three families of the standard model. For this purpose it may be adequate to study fermions in vector-like representations of the electro-weak gauge group with a large bare mass term, rather than the conventional chiral fermions. The main reason for this is from the strong experimental constraints on the S parameter [2]. While experiments favor a negative value of S [3], a standard chiral doublet of heavy fermions (degenerate in mass) contributes to the S parameter as $1/6\pi$. On the contrary, for fermions in the vector-like representation of the electro-weak gauge group, a large bare fermion mass M completely changes the low energy properties of the heavy fermions. As a consequence of the decoupling theorem, heavy fermions' contribution to the oblique corrections of the standard processes are suppressed by $\frac{1}{M^2}$. Especially, their contribution to the S parameter is still positive definite but much smaller in magnitude than the ordinary chiral fermions. Furthermore the heavy fermion contributes to the vacuum expectation value of electroweak symmetry breaking as [4],

$$\delta(f_\pi^2) = \delta v^2 \simeq \frac{m^2 N_c}{2\pi^2}(\log\frac{\Lambda^2}{M^2}), \tag{1}$$

where where m is the mass generated by the Yukawa coupling and Λ is the cutoff scale of the effective theory. It is interesting to compare the above expression to that of the pion decay constant obtained in the QCD effective action approach [5], $f_\pi^2 = \frac{N_c}{4\pi^2}M_Q^2\ln(\frac{\Lambda_{QCD}^2}{M_Q^2})$, where M_Q is the constitute quark mass which is similar to m in our present discussion. We notice that if in the above Eq. (1) $m \sim O(v)$ then several of these heavy fermions would be enough to induce the electroweak symmetry breaking. Therefore if there is a strong attractive forces in the appropriate channel to cause the heavy fermion condensation then they may place the role similar to techniquarks in the technicolor model. This way of dynamical electroweak symmetry breaking, if possible, is remarkable. Contrary to the technicolor model, it avoids the dangerous low energy consequences which may contradict experiments. Also it can be demonstrated [6] that the composite Higgs boson's mass is proportional to the dynamically generated fermion mass and completely decouples from the bare one, even though the Higgs particle is "composed of" the heavy fermions. This is a consequence of symmetry and be model independent, at least in a system with second order phase transition.

Heavy fermions may have many other interesting role in physics beyond the standard model either. For example, they may be responsible for a dynamical generation of light fermion mass matrix [7]; they appear in the "vector–like extension" of the standard model [8]; they are natural consequences of many grand unification models, and of the super-symmetric preon model [9]. Therefore it is important to investigate the fundamental properties of the heavy vector-like fermions thoroughly.

There have been continuous interests in understanding the structure of the standard model at high energies, even up to Planck scale (see for example, [10–14] and the most recent review which contains many materials, Ref. [15]). A powerful tool is to use the renormalization group equations to trace the evolution of the coupling constant of the $\lambda\phi^4$ self-interaction of the Higgs particle. Assuming the standard model remains valid up to certain scale Λ, an upper bound (the triviality bound, obtained by requiring λ not to blow

up below Λ) of the Higgs boson mass, m_H, can be obtained. Meanwhile, requiring the stability of the electro-weak vacuum, we can also obtain a lower bound on m_h. For the later purpose, in principle one needs to consider the renormalization group improved effective potential [16] and require it be bounded from below. But in practice this turns out to be equivalent to the requirement that the Higgs self-interaction coupling constant λ does not become negative, below the given scale (see [10] and ref. therein). It is remarkable that for the given experimental value of the top quark mass (here we use $m_t = 174$GeV), there is an allowed range for the Higgs boson mass, 130GeV $\leq m_H \leq$ 200GeV [10], for which the standard model may remain valid up to Planck scale.

In this paper we devote to study heavy fermions' influence to the vacuum stability bound and the triviality bound on the Higgs boson mass. Furthermore, assuming that the Higgs boson is a composite particle, we use the method developed in Ref. [1] to estimate the range of the Higgs boson's mass[1]. We find that the top quark also place an important role in the compositeness picture and the composite Higgs boson can be viewed as a mixture of $\bar{t}t$ pair and heavy fermion pair. The larger the hierarchy is the more top quark content the composite Higgs boson contains and vise–versa.

We start with the following general Lagrangian for heavy fermions,

$$\mathcal{L} = \bar{Q}(i\not{\Delta}_d - M)Q + \bar{U}(i\not{\Delta}_s - M)U + \bar{D}(i\not{\Delta}_s - M)D + g_d\bar{Q}_L\phi D_R + g_u\bar{Q}_L\tilde{\phi}U_R$$

$$+ g_d'\bar{Q}_R\phi D_L + g_u'\bar{Q}_R\tilde{\phi}U_L + h.c. \;. \tag{2}$$

In above Q is the $SU(2)_W$ doublet and U and D are singlets with weak hypercharge Y_Q, Y_U and Y_D, respectively (with the selection rule $Y_U - Y_Q = Y_Q - Y_D = Y_\phi$). We assume they participate in strong interactions and are in fundamental representations of $SU(3)_C$. The subscript d (s) in the covariant derivatives denotes that the corresponding fermion is a SU$(2)_W$ doublet (singlet) and ϕ denotes the standard Higgs doublet. We further expect the Yukawa couplings to be of order 1. For simplicity we take all the bare fermion masses to be equal. Also we do not discuss the mixing between heavy fermions and the ordinary fermions here.

As is well known, because of the negative sign, fermions turn to destabilize the vacuum. After including heavy fermions the structure of our world changes drastically at high scales, even though vector-like fermions are essentially decoupling below their threshold. At scales much higher than the threshold whether the fermion field is chiral or vector-like does not make any qualitative difference. The only thing matters is the number of independent Yukawa couplings and their strength. The relevant one loop RGEs are listed as below[2],

$$16\pi^2\frac{d\lambda}{dt} = 24\lambda^2 + 12\lambda A - 6A' - (9g_2^2 + 3g_1^2)\lambda + \frac{9}{8}g_2^4 + \frac{3}{4}g_2^2g_1^2 + \frac{3}{8}g_1^4 \;, \tag{3}$$

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[1] This paper replaces and is an extension of Ref. [17].

[2] Due to a careless mistake, the Yukawa coupling RGEs given in Ref. [17] contain an error. The top quark effects were not considered correctly.

$$16\pi^2 \frac{dg_u}{dt} = \{\frac{3}{2}(g_u g_u^\dagger - g_d g_d^\dagger) + 3A - 8g_s^2 - \frac{9}{4}g_2^2 - 3(Y_Q^2 + Y_U^2)g_1^2\}g_u \ , \tag{4}$$

$$16\pi^2 \frac{dg_d}{dt} = \{\frac{3}{2}(g_d g_d^\dagger - g_u g_u^\dagger) + 3A - 8g_s^2 - \frac{9}{4}g_2^2 - 3(Y_Q^2 + Y_D^2)g_1^2\}g_d \ , \tag{5}$$

$$16\pi^2 \frac{dg_t}{dt} = \{\frac{3}{2}g_t^2 + 3A - 8g_s^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\}g_t \ , \tag{6}$$

where,

$$A = \mathrm{tr}\{g_u g_u^\dagger + g_d g_d^\dagger + g_u'(g_u')^\dagger + g_d'(g_d')^\dagger\} + g_t^2 \ , \tag{7}$$

$$A' = \mathrm{tr}\{(g_u g_u^\dagger)^2 + (g_d g_d^\dagger)^2 + (g_u'(g_u')^\dagger)^2 + (g_d'(g_d')^\dagger)^2\} + g_t^4 \ , \tag{8}$$

and

$$16\pi^2 \frac{dg_s}{dt} = (-7 + \frac{2}{3}(2N_Q + N_U + N_D)\theta)g_s^3 \ , \tag{9}$$

$$16\pi^2 \frac{dg_2}{dt} = (-\frac{19}{6} + 2N_Q\theta)g_2^3 \ , \tag{10}$$

$$16\pi^2 \frac{dg_1}{dt} = (\frac{41}{6} + 4(2N_Q Y_Q^2 + N_U Y_U^2 + N_D Y_D^2)\theta)g_1^3 \ , \tag{11}$$

where the trace doesn't sum over color space and g_u' and g_d' obey similar equations. In general these Yukawa couplings can be matrices in the flavor space if there are many heavy fermions, and g_t is the Yukawa coupling of the top quark ($g_t = \sqrt{2}m_t/v$). The symbols N_Q, N_U and N_D refer to the number of Q, U and D type of quarks, respectively. We use a simple step function $\theta = \theta(t - log(M/M_z))$ to model the heavy fermion threshold effects. All the Yukawa couplings in above renormalization group equations are understood as multiplied by θ. Applications using two loop RGEs in the standard model case and beyond was considered in Ref. [18] and it was found that the two loop effects are very small below Planck scale.

In the following qualitative discussion, we set $Y_Q = 1/6$, $Y_U = 2/3$ and $Y_D = -1/3$. For simplicity we take $N_Q = N_U = N_D$ ($\equiv N$) and all the Yukawa couplings (after the diagonalization of the coupling matrices) in the initial boundary conditions being identical [3]. In fig. 1 we plot the vacuum stability bound and the triviality bound on the Higgs mass as a function of the scale Λ for some typical values of the parameters of the heavy fermions. We see that the inclusion of heavy fermions drastically change the Standard model structure at high energies even though they decouple from the low energy world. They tighten the

[3]The 'up' and 'down' type quarks evolve differently because of different $U(1)_Y$ charge, however the isospin splitting is very small for the standard values of the hypercharge.

bound on the mass of the Higgs boson as a function of the cutoff scale Λ. Notice that (in terms of one loop renormalization equations) the upper line (triviality bound) and the lower line (vacuum stability bound) never meet each other. Because the upper line is drawn by requiring λ not to blow up and the lower line is drawn by requiring $\lambda \geq 0$. Between them is the ultra-violet unstable fixed point of λ, so the two lines get close to each other rapidly.

We now study the interesting possibility of considering the Higgs particle as a composite object of the heavy vector-like fermions. Applying the above renormalization group analysis to the composite model leads to some interesting results which we present below. We follow the method proposed by Bardeen, Hill and Lindner (BHL) [1] originally developed for the top quark condensate model. The basic idea of the BHL method is the following: Using the collective field method the four–fermi interaction Lagrangian can be rewritten into an effective Higgs–Yukawa interaction Lagrangian at the cutoff scale Λ. The effective Yukawa interaction Lagrangian is identical to the standard model at the cutoff scale Λ, but with vanishing wave function renormalization constant of the Higgs field ($Z_H = 0$) and vanishing Higgs self-coupling ($\lambda = 0$). Below Λ the model is equivalent to the standard model and therefore the coupling constants of the effective theory run according to the standard model renormalization group equations. However the vanishing of Z_H at the scale $\mu = \Lambda$ leads to the following boundary conditions of the renormalization group equations:

$$g_Y^r \to \infty \ , \quad \lambda^r/(g_Y^r)^4 \to 0 \ , \tag{12}$$

where λ^r and g_Y^r are the renormalized Higgs self-coupling and Yukawa coupling, respectively. With the renormalization group equations and boundary conditions, one can predict the mass of the Higgs boson and the fermion mass (or the Yukawa couplings) at the infra-red fixed point. In the present case, of course, the "standard model" often refers to the standard model plus heavy fermions and the "infra-red fixed point" value of g_Y refers to its value at the threshold.

The minimal top quark condensate model has already been ruled out by experiments. In order to generate the electroweak symmetry breaking scale v, the top quark mass is required to be at least as large as 218 GeV (corresponding to $\Lambda = 10^{19}$ GeV, i.e., Planck scale). The experimental value of the top quark mass indicates that the top quark Yukawa coupling does not diverge up to Planck scale in the standard model and therefore does not meet the compositeness condition of BHL. This can be clearly seen from fig. 2. However, in the present model, since there is no strict experimental constraint on the heavy fermions, the compositeness condition is easily and naturally achievable, that g_t blows up below Planck scale with the aid of the heavy fermions. From Eqs. (4), (6) we see that the evolution of the Yukawa couplings are correlated to each other and one 'blows up' leads the another to blow up too.

When both the top quark and heavy fermions are involved, the situation is more complicated than the simple top condensate model. Running the RGEs down from certain scale, one must take good care of g_t to ensure that it reaches the experimental value at the infra-red fixed point. This means that a certain fine-tuning is needed on the initial boundary conditions of the Yukawa coupling RGEs. The composite Higgs boson is now a mixture of $\bar{t}t$ pairs and the heavy quark pairs. Fig. 3 and fig. 4 show two typical examples of such a situation. In the situation of fig. 3 the Higgs particle is mainly composed of heavy fermions while in fig. 4 the top quark becomes the major component. Notice that for a given ratio of

g_Y/g_t in the compositeness boundary condition (for fixed M and N), the composite scale Λ is no longer free, rather it is determined by g_t^{exp}.

In fig. 5 we plot the composite Higgs particle's mass[4] as a function of the composite scale, Λ. We chose $N \leq 3$ to avoid the problem with the non-asymptotic freedom of g_s. From fig. 5 we see that the allowed range for the Higgs mass is rather narrow against the wide range of the cutoff scale, the bare fermion mass and the number of heavy fermions, except when the heavy fermion bare mass M is close to the cutoff Λ. A lower bound on the Higgs mass can be obtained: $m_H \geq 150$ GeV. When M is getting close to the cutoff scale our results become unstable and are sensitive to the input numerical values of the boundary conditions. In such a situation the scale is not large enough for the couplings to reach the infra-red stable point. It is estimated that the Higgs mass will not exceed 450 GeV, otherwise the whole mechanism become unnatural (in the sense that the Yukawa coupling constant at electroweak scale also becomes substantially larger than 1).

In fig. 6 we plot a typical example of the Higgs mass for a given cutoff scale Λ_c and N. We also plot the triviality bound and the vacuum stability bound using the value of the Yukawa coupling constant at the infrared fixed-point, which is determined uniquely by the parameters M, Λ_c and N in the compositeness picture, as the initial boundary condition. It is very interesting to notice that m_H and Λ take the values where the curves of triviality bound and vacuum stability bound (practically) meet each other. This is the unique feature of BHL compositeness picture. The reason behind this is very simple: The infra-red attractive fixed point corresponds to the ultra-violet unstable fixed point. In the sense of Ref. [19], this picture can be disturbed. However in most cases the infra-red–ultra-violet fixed point structure is influential and rather stable against perturbation.

In above we presented an analysis on the properties of heavy fermions in vector-like representations of the standard model gauge group. We pointed out earlier [6] that if they can place the role to break the electro-weak symmetry dynamically the theory has some distinguishable properties: the low energy theory is asymptotically renormalizable and returns to the standard model. From the above RG analysis we realize that the top quark also places an important role in the dynamical symmetry breaking scenario and our model can be viewed as a natural generalization to the top condensate model of BHL. We found that the composite Higgs boson's mass ranges from 150GeV to 450GeV, and the lighter the Higgs boson is the more top quark content it contains, and vice versa. Our prediction to the mass of the Higgs boson will be testable by LHC and the model will be ruled out if m_H is found to be below 150GeV.

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[4] The Higgs mass in these figures is the renormalized mass at $\mu = M_Z$. The renormalized mass is close to the pole mass of the Higgs boson.

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FIGURES

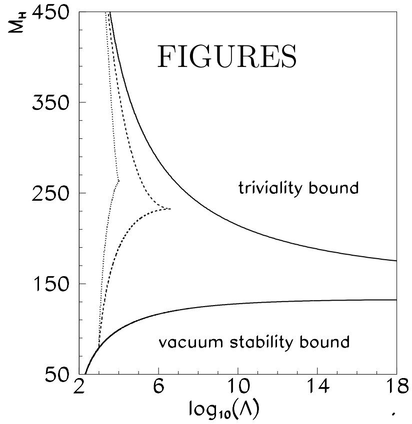

FIG. 1. Vacuum stability and triviality bounds on the Higgs mass as a function of Λ. The solid lines are the standard model case The dashed (dotted) lines correspond to $N = 1$ ($N = 3$), the Yukawa coupling $g_Y = 1$.

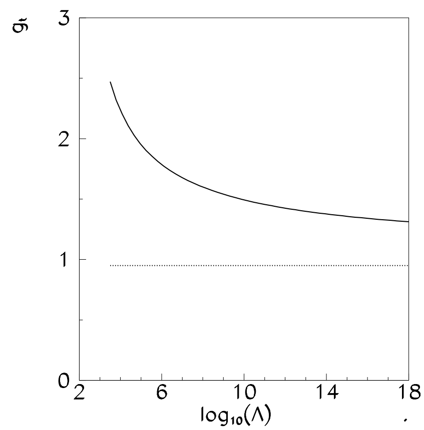

FIG. 2. The solid line: Infra-red fixed point value of g_t as a function of the compositeness scale according to the standard model RGEs. The dotted line indicates the experimental value of g_t.

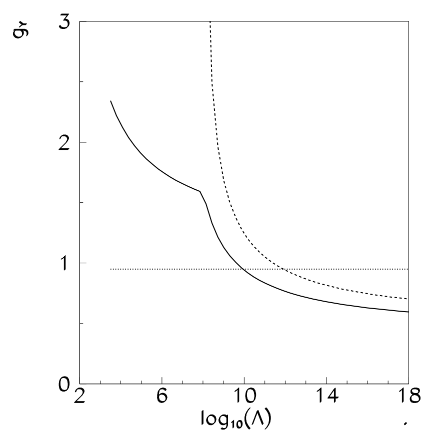

FIG. 3. Infra-red fixed point value of g_t (solid line) and g_Y (dashed line). The dotted line indicates g_t^{exp}. M=10^8 GeV, N=1. The correct value of the composite scale is at where the solid line cross the dotted line.

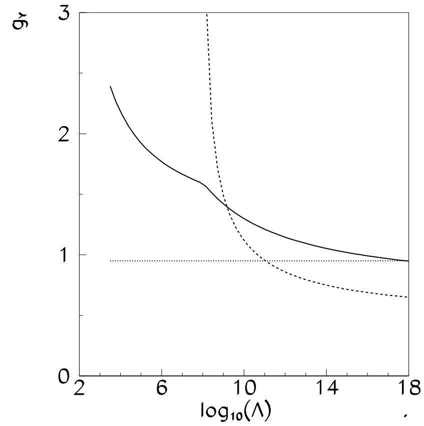

FIG. 4. Infra-red fixed point value of g_t (solid line) and g_Y (dashed line). The dotted line indicates g_t^{exp}. M=10^8 GeV, N=1. Here there are more top quark content in the composite Higgs boson.

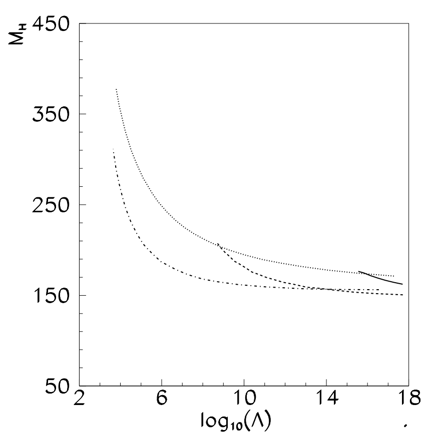

FIG. 5. IR fixed point value (at M_Z) of M_H as a function of the compositeness scale. The solid line: $N = 3$, $M = 10^{15}$ GeV; the dashed line: $N = 3$, $M = 10^8$ GeV; the dotted line: $N = 1$, $M = 10^3$ GeV; the dot-dashed line: $N = 3$, $M = 10^3$ GeV.

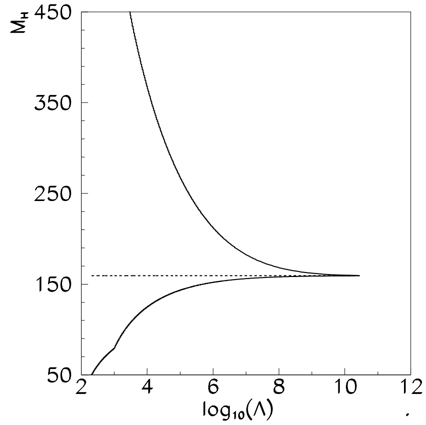

FIG. 6. IR–UV fixed point structure and compositeness. N=3, M=10^3 GeV, $\Lambda_C = 10^{11}$.